References

[1] Prosnak, W. J., "A note on the application of Pohlhausen's method to the stagnation point flow," Arch. Budowy Maszyn 10, (1963); also Intern. Aerospace Abstracts 4, 3–11 (1963).

[2] Hugelman, R. D. and Haworth, D. R., "An MHD boundary-layer compatibility condition," AIAA J. 3, 1367–1369 (1965).

[3] Schlichting, H., *Boundary Layer Theory* (McGraw-Hill Book Co., Inc., New York, 1960), 4th ed., pp. 137–140.

Approximate Expression for the Boundary-Layer Shape Factor

BRIAN L. HUNT* AND MERWIN SIBULKIN†
Brown University, Providence, R. I.

Introduction

THE momentum integral equation of boundary-layer analysis contains a shape factor, which is the ratio of displacement thickness to momentum thickness. With the aid of approximate velocity and temperature profiles, the shape factor may be obtained as a function of the external stream quantities and the profile parameters. Unfortunately, the resulting function generally is not analytical. This note considers the shape factor for the case where the velocity profile is given by a power law and the temperature profile by the Crocco relation; an accurate but simple analytical approximation to the shape factor is described.

Evaluation and Approximation of the Shape Factor

The boundary-layer shape factor H is defined by

$$H \equiv \frac{\delta^*}{\theta} \equiv \frac{\int_0^1 \left(1 - \frac{\rho u}{\rho_e u_e}\right) d\left(\frac{y}{\delta}\right)}{\int_0^1 \left(1 - \frac{u}{u_e}\right) \frac{\rho u}{\rho_e u_e} d\left(\frac{y}{\delta}\right)} \qquad (1)$$

where the subscript e denotes quantities at the external edge of the boundary layer.

In both laminar and turbulent compressible flows the velocity profile can be approximated by

$$u/u_e = (y/\delta)^{1/N} \equiv z \qquad (2)$$

When the Prandtl number of the fluid is unity and there is either no pressure gradient or no heat transfer from the wall, the Crocco relation holds. For a perfect gas this relation may be written

$$T^o - T_w = (u/u_e) - (T_e^o - T_w) = z(T_e^o - T_w) \qquad (3)$$

where T_w is the temperature at the wall and superscript o denotes stagnation values. Walz[1] suggests that Eq. (3) is also valid for flow over a nonadiabatic wall where the pressure gradient is moderate. From the definitions of Mach number and stagnation enthalpy it follows that

$$\frac{T}{T_e} = f(z;\ m, t) \equiv \frac{1+m}{t} + \frac{t-1}{t}(1+m)z - mz^2 \qquad (4)$$

Received February 27, 1965; revision received August 13, 1965. This work was supported by the Advanced Research Projects Agency (Ballistic Missile Defense Office) and technically administered by the Fluid Dynamics Branch of the Office of Naval Research under Contract Nonr 562(35).

* NATO Research Student.

† Associate Professor of Engineering.

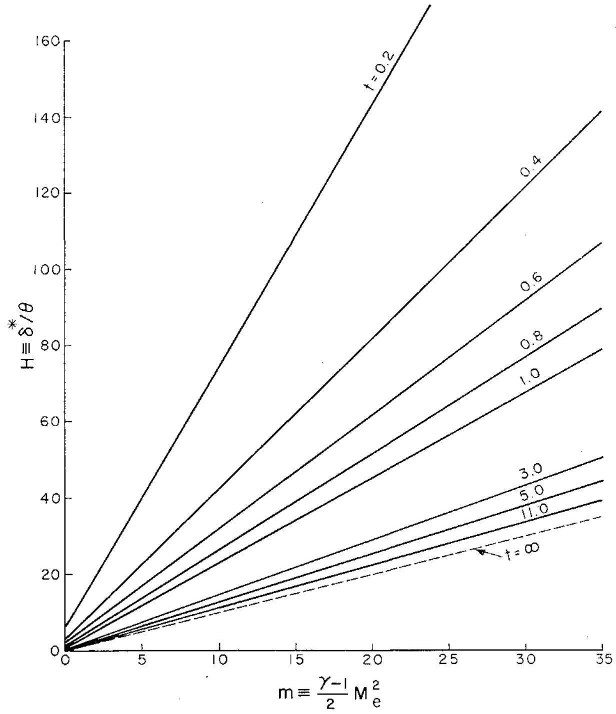

Fig. 1 The variation of shape factor with flow conditions when the velocity profile obeys a $\frac{1}{7}$ power law.

where $t \equiv T_e^o/T_w$ and $m \equiv [(\gamma - 1)/2]M_e^2$. Using Eqs. (2) and (4) and the perfect gas law, Eq. (1) may be rewritten in the form

$$H = H(m; t, N) = \frac{\int_0^1 \left[1 - \frac{z}{f(z;\ m, t)}\right] z^{N-1} dz}{\int_0^1 \left[\frac{1-z}{f(z;\ m, t)}\right] z^N dz} \qquad (5)$$

Bartz[2] has evaluated numerically the integrals in Eq. (5) for the case $N = 7$. Using Bartz's values, the function $H(m;\ t, 7)$ has been calculated and is shown in Fig. 1. As indicated by Fig. 1, the function is approximately linear in m for fixed values of t. The error in a linear representation never exceeds 2% and is less than 0.5% for $m > 16$. The integrals in Eq. (5) have been evaluated analytically for the case of $N = 1$ (Ref. 3), and the values of H again lie almost exactly on a family of straight lines. Tucker[4] has evaluated H for $t = 1$ and a number of values of N in the range 5 to 11. The values of $H(m;\ 1, N)$ again turn out to be approximately linear in m. Finally, the limit $N \to \infty$ of Eq. (5) has been taken (see Ref. 3 for details), leading to the result

$$H(m;\ t, \infty) = 1/t + (1 + 1/t)m \qquad (6)$$

which is exactly linear in m for fixed t.

It may therefore be concluded that a good approximation for H is

$$H(m;\ t, N) = H_0(t, N) + H_1(t, N)m \qquad (7)$$

Fitted values of H_0 and H_1 for $n = 1$ and 7 and analytical values of H_0 and H_1 for $N = \infty$ are shown in Figs. 2 and 3 together with values for $N = 5$ and $N = 11$ at $t = 1$.

The shapes of the curves in Figs. 2 and 3 suggest that H_0 and H_1 may be asymptotic to 0 and 1, respectively, for large t. This can be shown to be true by rearranging Eq. (1) into the form

$$H = \frac{\int_0^1 \left\{\left[\frac{(1+m)/t + (mt - 1 - m)z/t - mz^2}{f(z; m, t}\right] z^{N-1}\right\} dz}{\int_0^1 \left[\frac{(1-z)}{f(z;\ m, t)}\right] zN dz} \qquad (8)$$

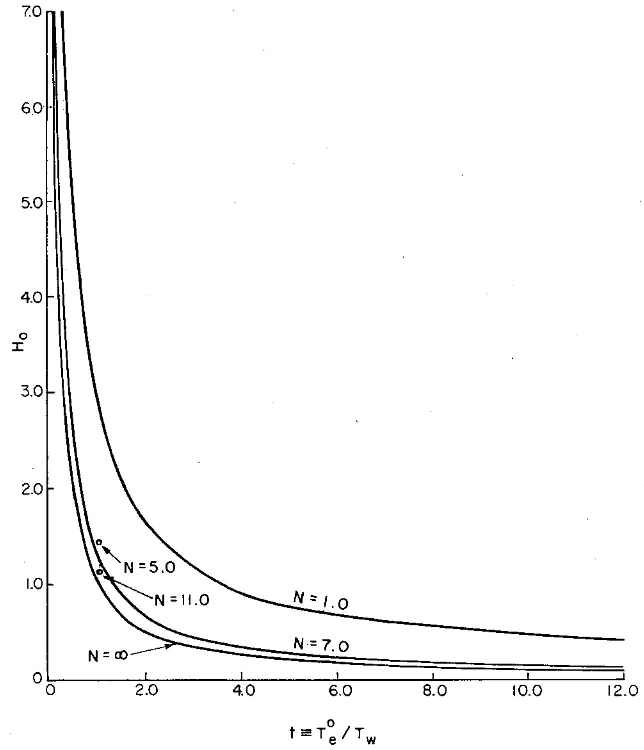

Fig. 2 The variation of H_0 with temperature ratio for different velocity profiles.

Taking the limit as $t \to \infty$ gives

$$H(m;\ \infty, N) = m \tag{9}$$

and this limiting equation for H has been included in Fig. 1. For $t \ll 1$

$$f(z) \approx (1 + m)(1 - z)/t \tag{10}$$

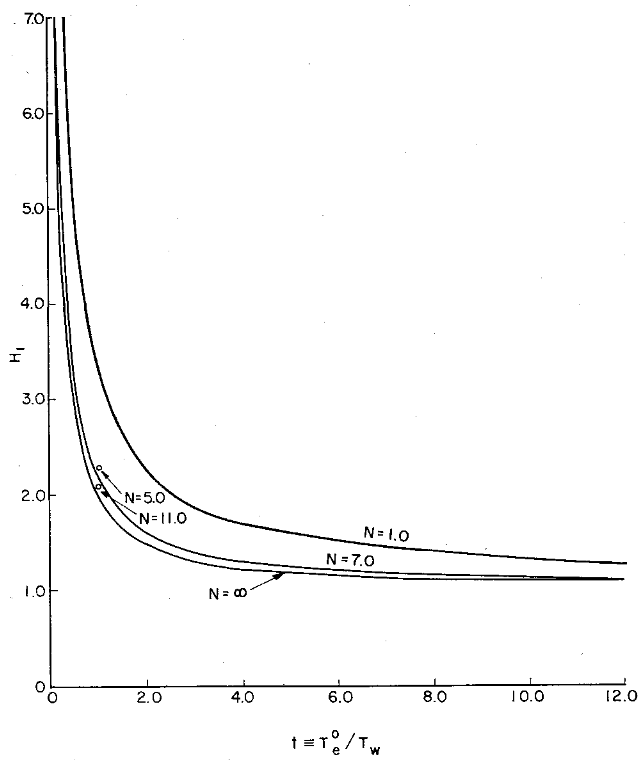

Fig. 3 The variation of H_1 with temperature ratio for different velocity profiles.

and Eq. (5) may be integrated to give

$$H \approx [(N + 1)/Nt](1 + m) \tag{11}$$

which also exhibits the suggested linearity in m.

Thus Figs. 2 and 3 and Eq. (9) show that at moderately large values of t (as found in hypersonic flow), H is almost independent of t. In turbulent flow, N generally lies in the range 5 to 11 and for compressible laminar flow, $N = 1$ is a reasonable approximation. Figures 2 and 3 show that, except at low values of t, change of N has only a small effect on H_0 and H_1 with laminar flow (low N) being more sensitive than turbulent flow (high N).

References

[1] Walz, A., "Compressible turbulent boundary layers," Mécan. de la Turbulence, Colloques Internationaux du Centre National de la Recherche Scientifique, Sec. No. 108, 299–350 (1961).

[2] Bartz, D. R., "An approximate solution of compressible turbulent boundary-layer development and convective heat transfer in convergent-divergent nozzles," Jet Propulsion Lab. Progress Rept. 20–234 (1954).

[3] Hunt, B. L. and Sibulkin, M., "An estimate of compressible boundary layer development around a convex corner in supersonic flow," Div. Engineering, Brown Univ., Rept. Nonr 562 (35)/6 (1964).

[4] Tucker, M., "Approximate turbulent boundary-layer development in plane compressible flow along thermally insulated surfaces with application to supersonic-tunnel contour corrections," NACA TN 2045 (1950).

Experiments on a Faraday-Type MHD Accelerator with Series-Connected Electrodes

K. E. Tempelmeyer,* L. E. Rittenhouse,† and D. R. Wilson‡
ARO, Inc., Arnold Air Force Station, Tenn.

Introduction

SINCE a conventional segmented-electrode magnetohydrodynamic (MHD) accelerator or generator develops an axial Hall electric field, the potential on a given anode may be equal to the potential on some preceding cathode in an accelerator or some following cathode in a generator. With this in mind, de Montardy[1] has suggested connecting the electrodes of a generator in series as illustrated in Fig. 1, thereby making it a two-terminal power supply. In order to simplify the external connections, Dicks[2] has extended this idea by pointing out that the sidewalls in MHD generators or accelerators might be made up of metallic strips that lie approximately along the equipotential lines and connect anodes and cathodes that have the same potential. These diagonal metal strips in the walls would replace the jumpers shown in Fig.

Received May 3, 1965; revision received July 12, 1965. Research reported in this paper was sponsored by the Arnold Engineering Development Center (AEDC), U. S. Air Force Systems Command, under Contract No. AF 40 (600)-1000 with ARO, Inc. Further reproduction is authorized to satisfy needs of the U. S. Government.

* Supervisor, MHD Section, LORHO Branch, Propulsion Wind Tunnel Facility; now Manager, Research Branch, Aerospace Environmental Facility, Associate Fellow Member AIAA.

† Research Engineer, MHD Section, LORHO Branch, Propulsion Wind Tunnel Facility; now Supervisor, MHD Section, Member AIAA.

‡ Research Engineer, MHD Section, LORHO Branch, Propulsion Wind Tunnel Facility.